Aerodynamic characteristics of a multi-copter（2017）

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List of Main Symbols

Symbol	Explanation	unit
M:	Vehicle dry weight	kg
S:	Frontal projected area	m^2
α:	Pitch angle	rad
$\dot{m}$:	Air mass flow	kg/s
v_i:	Induced velocity	m/s
V_∞:	Uniform flow velocity	m/s
T:	Thrust	N
L:	Lift	N
D:	Drug	N
P:	Power	W
A:	Rotor disk area	m^2
C_T:	Thrust coefficient	-
C_P:	Power coefficient	-
C_Q:	Torque coefficient	-
C_D:	Drug coefficient	-
FM:	Figure of merit (hover)	-
η:	Figure of merit (forward flight)	-
Q:	Torque	Nm
ω:	Rotational speed	rad/s
R:	Rotor radius	m
E:	Voltage output	V
I:	Current output	A
g:	Acceleration of gravity	m/s^2
ρ:	Air density	kg/m^3
q:	Dynamic pressure	Pa

Definition of Symbols

$$C_T \equiv \frac{T}{\rho A \omega^2 R^2}$$

$$C_P \equiv \frac{P}{\rho A \omega^3 R^3}$$

$$C_Q \equiv \frac{Q}{\rho A \omega^2 R^3}$$

1. Course purpose

Recently, the multi-copter has been attracting the widespread interests as a usage of aerial photography, structure inspection, and 3-D mapping. The currently prominent multi-copters make a flight with the benefits of the high response electrical motors, and feature the high positional and attitude stability. Although these excellent flight abilities have been realized by the advanced control technics inside the Flight controller, the careful attention has not been paid to the aerodynamics ability of the multi-copter. In fact, there have been several fall and crash accidents on and off. To realize the stable and safe flight of a multi-copter, it is important to investigate the physical flight criteria. In this experiment, the flight data such as attitude is measured by the performance of the flight test, using Phantom4 (Fig.1) made in DJI corporation. Then, aerodynamics coefficient is measured through the the airspeed. Furthermore, comparing with helicopter and fixed wing aircraft, the understanding of aerodynamics characteristics of multi-copter was expected to be deepen.

Fig.1：Phantom4 made in DJI company[1]

2. DJI Phantom4

Phantom 4 is developed specifically for aerial photographs. Multiple sensors equipped with Phantom4 enables extremely high stability and redundancy. In addition, the data measured and calculated by the sensor are recorded in the logger inside the aircraft, and by extracting these flight logs, various physical quantities necessary for calculating aerodynamic characteristics can be measured.

2．1 attitude

For multi-copter, sensors such as acceleration, gyroscope, and geomagnetic sensors are used to estimate the attitude angle. Each sensor is fixed to the fuselage and measures the acceleration, angular acceleration, and geomagnetic intensity around the fuselage fixed coordinates (X, Y, Z axis), respectively. This method is called a strap-down method (for referring to the direction of gravitational acceleration), and it is used in UAV like multi-copter, satellite, rocket and so on. The measured data is integrated by certain filters such as a Kalman filter, and the attitude angle is calculated from several data.

Meanwhile, in passenger aircrafts and submarines, a mechanical gyroscope (earth sesame) is used to create a plane (platform) parallel to the earth, and the attitude angle is measured.

Although the strap down system is small and lightweight, it has a large amount of calculation for attitude angle estimation. Under these circumstances, it has become popular with the development of electronic computers in recent years.

In this experiment, the attitude angle is calculated by AHRS (Attitude Heading Reference System, Fig. 2) in Phantom 4.

Fig.2：AHRS inside Phantom4 [2]

2．2 motor

Phantom 4 uses a brushless motor for flight. Brushless motors can be modeled like DC motors.

In the DC motor, the following relational equation is established between the current I and the output torque Q.

$$Q = K_T I \tag{1}$$

Although the value of K_T in the ideal rotor is supposed to have the constant value, that is not true of the real rotor and the value K_T is changed due to several causes.

In this experiment, K_T was measured beforehand with the same rotor of Phantom4 and the output of motor torque is estimated during the flight as shown in Fig.3 and Table 1.

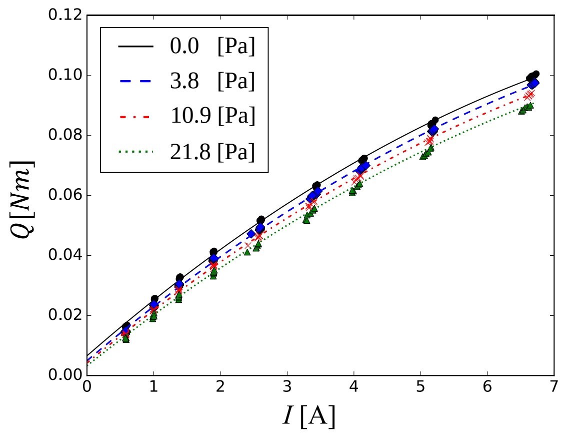

Fig.3：The relationship between current I and torque Q at each dynamic pressure.

Table 1：Interpolation formula between current I and torque Q at each dynamic pressure.

q[Pa]	Interpolation formula
0.0	$Q = -8.25\cdot10^{-4}\cdot I^2 + 1.94\cdot10^{-2}\cdot I + 6.59\cdot10^{-3}$
3.8	$Q = -7.63\cdot10^{-4}\cdot I^2 + 1.88\cdot10^{-2}\cdot I + 4.95\cdot10^{-3}$
10.9	$Q = -7.29\cdot10^{-4}\cdot I^2 + 1.83\cdot10^{-2}\cdot I + 4.20\cdot10^{-3}$
21.8	$Q = -6.87\cdot10^{-4}\cdot I^2 + 1.77\cdot10^{-2}\cdot I + 3.28\cdot10^{-3}$

2．3　Position control

Although a position control of a drone has widely used GPS system, it is impossible to perform the position control of a drone under the environment such as indoor where a Radio wave shielding occurs. Recently, with a hope to make it possible to estimate the accurate position control, other kinds of position control technics that utilize WiFi wave or the image recognition have been in practical use. Phamtom4 we use in this experiment has mounted the sensor technic that estimates the ground speed with the image recognition (optical sensor), and be able to perform the advanced position control even indoors.

3．rotor

Rotor performances can be roughly estimated by a change of momentum (**Momentum Theory**). The theoretical limitation of the rotor performance is provided in Momentum Theory.

In addition, more detailed analysis can be done by integrating the thrust and power calculated at each part of the rotor along the span direction. (**Blade Element Analysis**)

3．1　Hover

Considering momentum change before and after the rotor, equation (2) is derived. Furthermore, considering the work made into the fluid around the rotor, equation (3) is derived. (See separate explanation sheet)

$$v_i = \sqrt{\frac{T}{2\rho A}} \qquad (2)$$

$$P = \sqrt{\frac{T^3}{2\rho A}} \qquad (3)$$

Equation (3) gives the minimum value of power on the condition where the thrust T was performed in the rotor of disk area A. In an actual rotor, additional power is required from the value of the expression (3) due to harmful drag. There is an index (FM :Figure of Merit) for evaluating the performance of the rotor during hovering, and it is defined in equation (4).

$$\mathrm{FM} \equiv \frac{P_{\mathrm{ideal}}}{P_{\mathrm{mesure}}} = \frac{1}{P}\sqrt{\frac{T^3}{2\rho A}} = \frac{C_{\mathrm{T}}^{3/2}}{\sqrt{2}C_{\mathrm{P}}} \qquad (4)$$

3．2　Forward Flight

The flow model of a forward flight was shown in Fig.4.

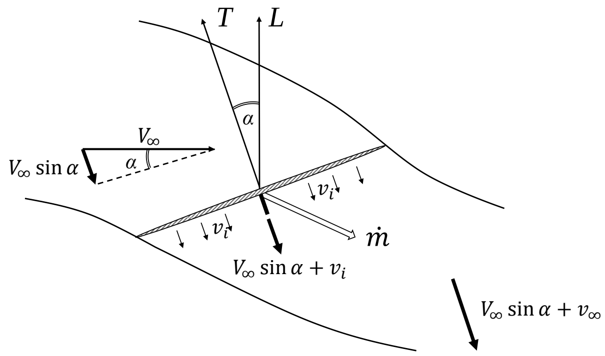

Fig.4：A change of the momentum in forward flight

The mass flow rate, $\dot{m}$, through the disk is now

$$\dot{m} = \rho A\sqrt{(V_\infty \cos\alpha)^2 + (V_\infty \sin\alpha + v_{\mathrm{i}})^2} \qquad (5)$$

The thrust T is expressed in the equation (6) by considering the momentum conservation in the vertical direction in the rotor.

$$T = 2\rho A v_{\mathrm{i}}\sqrt{(V_\infty \cos\alpha)^2 + (V_\infty \sin\alpha + v_{\mathrm{i}})^2} \qquad (6)$$

At the time of forward flight, the rotor performs $(V_\infty \sin\alpha + v_i)T$ work on the fluid. For this reason, η defined by equation (7) is used for performance evaluation of the rotor.

$$\eta \equiv \frac{(V_\infty \sin\alpha + v_{\mathrm{i}})T}{P} \qquad (7)$$

Note that v_i of equation (7) was the value deduced from the equation (6).

4．Methodology

4．1　**Preparation for the experiment**

1．**Measurement of the gravity center of a drone**

Although it is necessary to tilt the aircraft at the time of forward flight, a moment is generated since the gravity center does not coincide with the wind pressure center. In order to calculate the moment coefficient C_M, the position of the gravity center is measured.

The measurement procedure of the gravity center is shown below.

1) Hang the Phantom 4 with a string at different positions and take a picture.

2) Since the gravity center lies on the extension of the string, images under different hanging positions are superimposed to calculate the center of gravity.

3) Calculate the position of the gravity center by referring to pre-known values such as distance between rotors.

4．2　Flight Test

Flight tests are divided into two types, 1. hovering test and 2. forward flight test.

1．Hovering test

In the hovering test, drone is made to hover around 1.5 m in the air. Make a note of the installed payload and acquire data of various payloads. The experimental procedure is shown below.

1) Make sure there are no obstacles within 5 m radius of the place to fly.

2) Equip Phantom4 with the specified payload.

3) Turn on the Phantom4 and a controller.

4) After flying, hover for over 10 seconds at a height of about 1.5 m.

5) Extract flight data from Phantom 4.

2．　Forward Flight

In the forward flight test, the drones are hovered downstream of the low speed wind tunnel, simulating the time of forward flight. Data is acquired when the dynamic pressure of the wind tunnel is set to 3.8 Pa, 10.9 Pa, 21.8 Pa. Dynamic pressure is acquired by Pitot tube. Pitot tube is a device to measure differential pressure between total pressure and static pressure. From Bernoulli's principle, the relation between the dynamic pressure and the flow velocity is as shown in equation (9).

$$q = \frac{1}{2}\rho V_\infty^2 \tag{9}$$

The experimental procedure is shown below.

1) Make sure there are no obstacles within 5 m radius of the place to fly.

2) Adjust the dynamic pressure of the wind tunnel to achieve the specified dynamic pressure.

3) Turn on the Phantom4 and the controller.

4) After flying, make it fly in the downstream of the wind tunnel and hover for 10 seconds or more.

5) Extract flight data from Phantom 4.

5．Arrangement of experimental data

1．Data selection

All the flight data of Phantom4 was logged during operation. Since the data logged inside the Phantom4 include the unnecessary data for the experiment, data are separated between the necessary data and the unnecessary data. In addition, since the steady flight is only considered in this experiment, we search for the high steady sections of data and analyze data by the average value through that sections. (Fig.7)

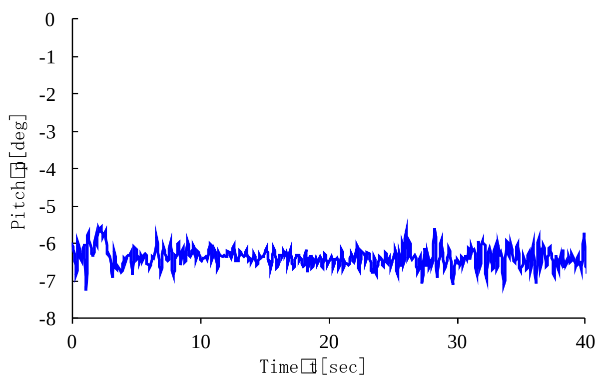

Fig.7：An example of steady flight data(pitch angle)

2．The treatment of C_T in forward flight

To analyze the data in forward flight, thrust ratio between the front and rear rotor needs to be given. Strictly, since airflow is generated along the rotor in forward flight, the value of dynamic pressure and attack of angle changes and the rotational speed of rotor comes to depend on C_T. However, like this experiment, when a forward speed of the vehicle is small enough compared to rotor rotational speed, fluctuation of C_T regarded to be negligible and is calculated assuming that C_T does not depend on rotor rotational speed. (Fig.8)

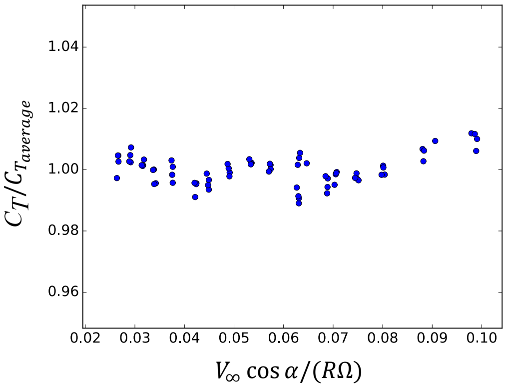

Fig.8 : the fluctuation value of measured C_T in this experiment [3]

6. Report

Report 1 Discussion on aerodynamics coefficient

1. Hover test

1) Derive the equations of (5) and(6). In addition, discuss what the equation (3) corresponds to in a fixed wing.

2) Calculate C_T, C_P, making use of characteristic data of motor (Fig.4, Table 1) and the data extracted from Phantom4 (download in HP). In addition, calculate FM from C_T, C_P and discuss the result.

3) Perform the same analysis as a question 2) in the case of changing the payload weight in a step-by-step manner.

2. Forward flight test

4) Calculate C_T by considering the vertical force balance. In addition, calculate C_P.

5) Calculate η at each dynamic pressure and plot that. In addition, explain the distribution of calculated η.

6) Calculate and plot C_D and C_M at each wind speed, considering the force and moment balance at steady flight. Frontal projected area at an attack of angle 0° can be used as a representative area S , and the distance between the rotors as a representative length L.

$$C_D = \frac{D}{qS} \tag{10}$$

$$C_M = \frac{M}{qSL} \tag{11}$$

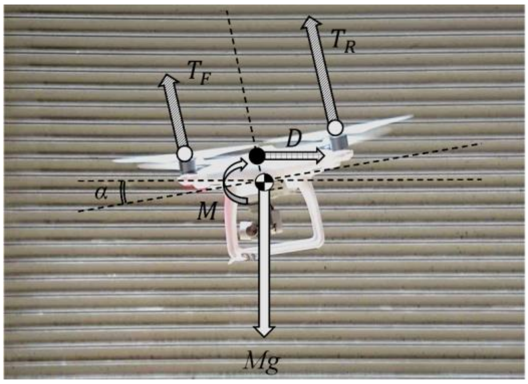

Fig.9 : Force and moment at forward flight

Report 2 Discussion on the performance of multi-copter

7) State the difference of a thrust changing method between a typical helicopter and a drone. In addition, explain the pros and cons of them from the reference literature.

8) Estimate the maximum flight speed of Phantom4. Note that C_D, C_M, η can be used from the value calculated in Report 1.

9) Discuss the optimal battery weight of Phantom4 and consider how much the hovering time can be improve by the change of the battery.

7. Reference material

Table 2 : Phantom4 specification

Body weight[g]	918
Rotor radius[m]	0.12
Disk area[m^2]	0.04524
Frontal projected area[m^2]	0.185
Motor maximum output[W]	80
Battery weight[g]	462
Battery energy density [kJ/kg]	633

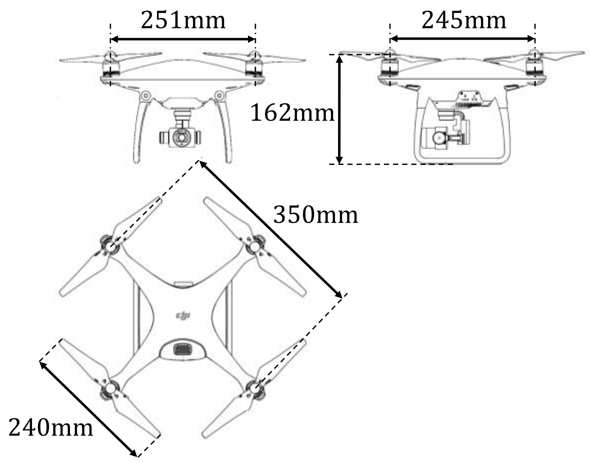

Fig.10：Phantom4 measurement

8. Annotation

[1], [2] DJI company HP(www.dji.com/phantom-4/)

[3] Measurement result at α =[0,3,6,9,12] unit : deg) and q=[0,3.8,10.9,21.8] (unit :Pa)